Limit Laws

Suppose $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ both exist. Then:

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$

3. $\lim_{x \to a} [c\, f(x)] = c \lim_{x \to a} f(x), \quad c \in \mathbb{R}$ a constant.

4. $\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

5. $\lim_{x \to a} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}, \quad$ provided $\lim_{x \to a} g(x) \neq 0$

6. $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x) \right]^n, \quad n \in \mathbb{N}$

7. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}, \quad n \in \mathbb{N}$

8. $\lim_{x \to a} c = c, \quad c \in \mathbb{R}$ a constant.

9. $\lim_{x \to a} x = a$

10. $\lim_{x \to a} (x^n) = a^n, \quad n \in \mathbb{N}$

These apply to one-sided limits as well; for example:

$$\lim_{x \to a^+} [f(x) + g(x)] = \lim_{x \to a^+} f(x) + \lim_{x \to a^+} g(x)$$