Homework 1, due Wednesday, 1/22:

1. Prove that the operation $*$ defined on $\mathbb{Q} - \{0\}$ by $a * b = \dfrac{a}{b}$ is not associative.

2. Prove that the operation $*$ defined on $\mathbb{Q}$ by $a*b = a+b+ab$ is associative.

3. Suppose $x, y, z \in G$ where G is a group (under the operation of multiplication). What is the inverse of the element xyz?

4. Suppose G is a group (under the operation of multiplication) with identity 1 such that $x^2 = 1$ for every $x \in G$. Prove that G is abelian.

5. Let $G = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$.

(a) Prove that G is a group under addition.

(b) Prove that $G - \{0\}$ is a group under multiplication.

[For both parts, don't forget to show that the operation is in fact a binary operation on G – i.e. $x, y \in G$ implies $x * y \in G$. Also (for both parts), you can save yourself a lot of trouble if you argue that associativity is inherited from $\mathbb{R}$ (since $G \subset \mathbb{R}$, $x, y, z \in G$ implies $x, y, z \in \mathbb{R}$).]