Homework 2, due Friday, 1/31:

1. Let H and K be subgroups of G. Prove that $H \cap K$ is a subgroup of G.

2. Let H and K be subgroups of G. Prove that $H \cup K$ is a subgroup of G if and only if either $H \subseteq K$ or $K \subseteq H$.

3. Let a be an element of a group G. Prove that $C_G(a) = C_G(a^{-1})$.

4. Prove that if $H \leq G$, $K \leq G$, and $H \subseteq K$, then $H \leq K$.

5. Give an example of an infinite subset H of an infinite group G such that H is closed under the group operation (i.e. $h_1, h_2 \in H$ implies $h_1 * h_2 \in H$), but H is *not* a subgroup of G.

6. Let G be a group.

   (a) Let $H \leq G$. Show that $C_G(H) = \{g \in g \mid g^{-1}hg = h \text{ for all } h \in H\}$.

   (b) Let $g, h \in G$. Prove that $(g^{-1}hg)^n = g^{-1}h^n g$ for any $n \in \mathbb{N}$.

   (c) Conclude that $C_G(h) \leq C_G(h^n)$ for any $n \in \mathbb{N}$. (In other words, *if g commutes with h, then g commutes with all powers of h.*)