

**Homework 5, due Friday, March 6th:**

1. Let  $\varphi : G_1 \rightarrow G_2$  be a homomorphism, and let  $H_1 \leq G_1$ .

(a) Prove that  $\ker \varphi \leq G_1$ .

(b) Prove that  $\varphi(H_1) \leq G_2$ .

(c) Prove that if  $H_1$  is abelian, then  $\varphi(H_1)$  is abelian.

[Hint: Let  $a, b \in H_1$ , and observe that  $ab = ba$  implies  $a^{-1}b^{-1}ab = 1$ . Then consider  $\varphi(a^{-1}b^{-1}ab)$ .]

[Note that, since  $G_1 \leq G_1$ , your results in parts (b) and (c) imply that  $\varphi(G_1) \leq G_2$ , and if  $G_1$  is abelian, then  $\varphi(G_1)$  is abelian.]

2. (a) Let  $Z_n = \langle x \rangle$ . Prove that the mapping  $\varphi : \mathbb{Z} \rightarrow Z_n$  given by  $\varphi(k) = x^k$  for any  $k \in \mathbb{Z}$  is a homomorphism.

(b) Find the kernel and the image of the homomorphism in part (a).

(c) Use the *First Isomorphism Theorem* to prove that  $\mathbb{Z}/n\mathbb{Z} \cong Z_n$ .